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# Combating Human Trafficking Using Mathematics

Alfred (AJ) Vogt  
*Duquesne University*

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## **Combatting Human Trafficking Using Mathematics**

**AJ Vogt, McAnulty College and Graduate School for Liberal Arts**

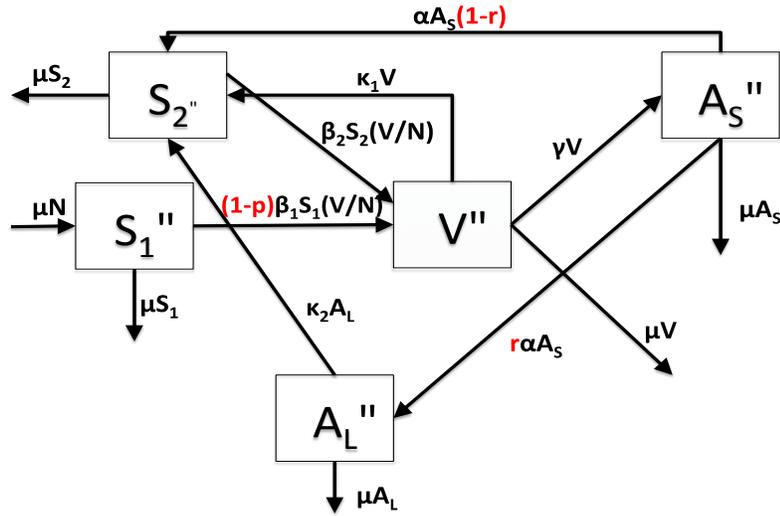
**Faculty Advisor: Rachael Miller Neilan, Ph.D.**

### **Introduction**

Human traffickers use force, fraud or coercion to lure their victims into labor or commercial sexual exploitation. Human trafficking occurs on a daily basis throughout the United States with the majority of victims being women and children. Nearly \$9.5 billion is generated annually in the U.S. through trafficking of humans (Wheaton et. al., 2010) and crime experts predict human trafficking will surpass drug and arms trafficking in its incidence, cost to human wellbeing, and profitability within the next ten years (Wheaton et. al., 2010). According to the National Center for Missing & Exploited Children (NCMEC), 1 in 6 runaways reported to NCMEC were likely involved in child sex trafficking (“Child Sex Trafficking,” 2016). Often times, human trafficking victims do not come forward because of language barriers, fear of their traffickers or fear of law enforcement.

This study aims to evaluate the costs and benefits of programs that prevent human trafficking through education and programs that provide long-term aid for victims of human trafficking. Prevention programs include educating children, teachers, community members, and law enforcement about human trafficking. Education can occur via training programs, both in person and through visual aids such as brochures and pamphlets. For example, in the Pittsburgh Metro area, the Human Trafficking Coalition of Pittsburgh organizes educational outreach events with the focus on preventing human trafficking (“The Project to End Human Trafficking,” [endhumantrafficking.org](http://endhumantrafficking.org)). On the other hand, long-term aid programs target services to previously victimized individuals. Long-term aid often occurs at facilities that provide housing, case management, emotional and psychological support to victims. Some long-term aid facilities provide education since the majority of victims are underage and do not have a high school education. An example of a long-term aid facility is the Wildflower Ranch in Denver, Colorado.

## Model Description



**Figure 1:** Diagram of mathematical model of human trafficking

We propose a mathematical framework for analyzing the costs and benefits of human trafficking prevention and long-term aid programs among a population of underage females. Figure 1 illustrates the general dynamics of the proposed model. Let  $N$  denote the size of the total population, i.e., females between the ages of 12 and 18 and living in poor socioeconomic conditions. The population is divided into five classes. Class  $S_1$  represents females who are susceptible to victimization and have not previously been the victim of human trafficking. Class  $S_2$  represents females who have previously been victimized and are susceptible to repeat victimization. Class  $V$  represents the victims of human trafficking. Finally, classes  $A_S$  and  $A_L$  represent previously victimized females who are currently receiving either short-term aid or long-term aid, respectively.

The arrows, in Figure 1, illustrate movement of individuals between the classes and into or out of the population. Susceptible females without prior trafficking incidences (class  $S_1$ ) become victims at the rate  $\beta_1 S_1 \left(\frac{V}{N}\right)$  where  $\beta_1$  is the per capita female trafficking recruitment rate and  $V/N$  is the probability of interacting with other trafficking victims. Similarly,  $\beta_2 S_2 \left(\frac{V}{N}\right)$  is the rate at which susceptible females with

prior trafficking incidences (class  $S_2$ ) become repeat victims. Here, we assume that  $\beta_2 > \beta_1$ . Human trafficking victims escape class  $V$  by leaving on their own and returning to susceptible class (class  $S_2$ ) at rate  $\kappa_1 V$  or by being rescued by law enforcement officials at rate  $\gamma V$ . Rescued victims are immediately placed into a short-term aid program (class  $A_s$ ). After receiving short-term aid, a female may receive additional long-term aid services (class  $A_L$ ). Those that do not receive additional services, return to class  $S_2$  at the rate  $\alpha A_s(1 - r)$  where  $\alpha$  is the rate of leaving short-term care and  $(1 - r)$  is the proportion of those opting to not go onto a long-term care facility. Those that opt to go to a long-term aid facility enter the  $A_L$  class at rate  $r\alpha A_s$ . It is intended that females in a long-term aid program remain in the program until they are no longer underage, however, it is possible that these females return to the susceptible population (class  $S_2$ ) at rate  $\kappa_2 A_L$ . Lastly, we assume a constant population size  $N$  over time so that  $\mu$  represents the rate of entrance into the population due to births as well as the rate of exit from the population due to aging.

The mathematical model represented in Figure 1 can be written as the system of five coupled differential equations seen below.

$$\frac{dS_1}{dt} = \mu N - \mu S_1 - (1 - p)\beta_1 S_1 \left(\frac{V}{N}\right) \quad (1)$$

$$\frac{dS_2}{dt} = \kappa_1 V + \kappa_2 A_L + \alpha A_s(1 - r) - \mu S_2 - \beta_2 S_2 \left(\frac{V}{N}\right) \quad (2)$$

$$\frac{dV}{dt} = (1 - p)\beta_1 S_1 \left(\frac{V}{N}\right) + \beta_2 S_2 \left(\frac{V}{N}\right) - (\kappa_1 + \gamma + \mu)V \quad (3)$$

$$\frac{dA_s}{dt} = \gamma V - \alpha A_s - \mu A_s \quad (4)$$

$$\frac{dA_L}{dt} = r\alpha A_s - \mu A_L - \kappa_2 A_L \quad (5)$$

## Parameters

Table 1: Description and value of all parameters used in model simulations.

Notation	Description	Value
$\mu$	Rate of exit and entrance of population	$1/6 \text{ year}^{-1}$
$\beta_1$	Recruitment rate of victims from $S_1$	$0.6 \text{ year}^{-1}$
$\beta_2$	Recruitment rate of victims from $S_2$	$1.2 \text{ year}^{-1}$
$\kappa_1$	Rate at which victims escape trafficker	$0.067 \text{ year}^{-1}$
$\kappa_2$	Rate at which victims leave long-term aid program	$0.067 \text{ year}^{-1}$
$\gamma$	Rate at which trafficked victims are rescued	$0.267 \text{ year}^{-1}$
$\alpha$	Rate of departure from short-term aid	$4 \text{ year}^{-1}$
$r$	Proportion of aided victims that continue to long-term aid	$0 \leq r \leq 0.45$
$p$	Reduction in victim recruitment rate due to prevention	$0 \leq p \leq 0.20$

Table 1 indicates the values used in simulations of our model. Many values were estimated from a similar model of the Sex Worker Industry (Davidoff et. al., 2006). Other values, such as  $\beta_2$ , were estimated based on knowledge of the human trafficking culture. For example,  $\beta_2$  was chosen to be double the value of  $\beta_1$  because previously victimized females are at greater risk for (repeat) victimization compared to females who have not been associated with human trafficking before. Rates of departure out of the population ( $\mu$ ) and out of short-term aid ( $\alpha$ ) were calculated using the average length of the time spent in each. Two parameters of particular interest are  $r$  (proportion of victims in short-term aid that continue to long-term aid) and  $p$  (reduction in victim recruitment rate due to prevention). These parameters were allowed to take any value in their assigned ranges so that the impact of different combinations of prevention and aid programs may be assessed. Realistic upper bounds on the ranges were assigned such that at most 45% of victimized females may enter the long-term aid program and at most a 20% reduction in victimization may be achieved through a prevention program.

Initial conditions below were chosen such that the population consists of 1000 females with 50 victims at the start of the simulation.

$$S_1(0) = 845, S_2(0) = 100, A_L(0) = 0, V(0) = 50, A_S(0) = 5 \quad (6)$$

## Simulations

For a variety of  $p$  and  $r$  values, we used Maple 17 to computationally solve system (1) – (5) with the initial conditions in (6). Solutions were found for time  $t = 0$  to  $t = 10$ . Additionally, we computed the total number of repeat victims as

$$\int_0^{10} \beta_2 S_2(t) \frac{V(t)}{N} dt,$$

the total number of first time victims as

$$\int_0^{10} (1 - p) \beta_1 S_1(t) \frac{V(t)}{N} dt,$$

and the total number of victims as

$$\int_0^{10} \beta_2 S_2(t) \frac{V(t)}{N} + \int_0^{10} (1 - p) \beta_1 S_1(t) \frac{V(t)}{N} dt.$$

Further, the total cost of a prevention program was calculated as

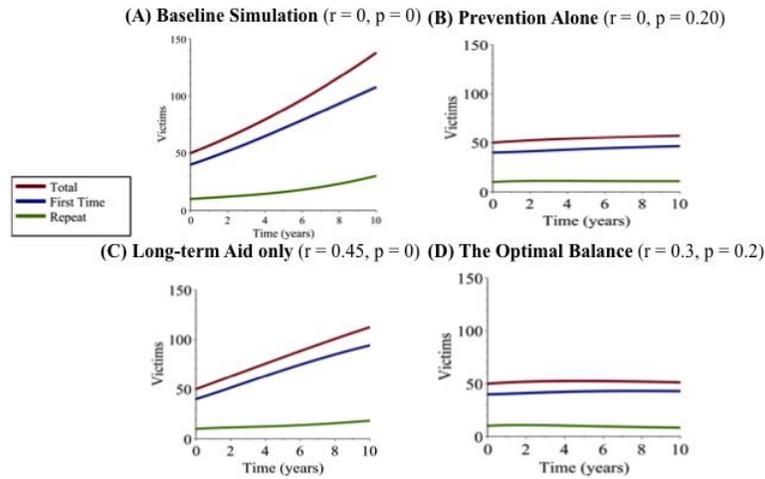
$$\int_0^{10} C_P (S_1(t) + S_2(t)) dt,$$

and the total cost of a long-term aid program as

$$\int_0^{10} C_L A_L(t) dt.$$

The constants  $C_P = 3,125p^2 + 187.50p$  and  $C_L = 14,000$  are the per person costs per year of a prevention program and long-term aid program, respectively.

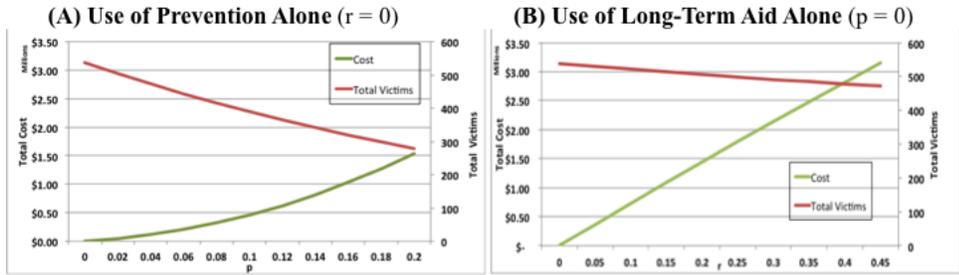
## Results



**Figure 2:** Model output showing the number of victims over time with different combinations of prevention and long-term aid.

Figure 2 displays the number of first time, repeat, and total victims over time outputted from four different simulations of our model. The baseline simulation (Figure 2A) shows the number of victims observed in the absence of any prevention or long-term aid program (i.e.  $p = 0$  and  $r = 0$ ). In this scenario, the initial quantity of 50 total victims triples over a ten-year period with repeat victims accounting for approximately 25% of total victims.

The results of using prevention alone or long-term aid alone are displayed in Figures 2B and 2C. In each of these simulations, the program was assumed to be at maximum capacity (i.e.  $p = 0.2$  or  $r = 0.45$ ). However, we explored the costs and benefits of each program alone when operating at different intensities. Figure 3A shows the costs of a prevention program and the number of resulting victims for different values of  $p$ . When the prevention program is operating at its highest capacity, the total number of victims is reduced by 49% from baseline and the cost is 1.5 million dollars over a ten-year period. Although prevention is a relatively cheap and effective method, it is limited in scope because it does not address the needs of victims or reduce the rate of repeat victimization. Figure 3B shows the costs



**Figure 3:** Total costs and total victims over a 10-year period for different applications of prevention or long-term aid alone.

of a long-term aid program and the number of resulting victims for different values of  $r$ . When the long-term aid program is operating at its highest capacity, the total number of victims is reduced by only 12% from baseline and the cost is 3.1 million dollars over a ten-year period. Although this is a more expensive method of victim reduction, it is essential in that it provides the tools necessary for victims to successfully return to their community and avoid repeat victimization. Using either prevention or long-term aid alone is not sufficient for addressing the complex problem of human trafficking. We used our model to explore different combinations of the two types of programs such that the total expenses did not exceed 3 million dollars over a ten-year period. This 3 million dollar budget is an example of a constraint imposed by a federal grant. Table 2 shows combinations of  $p$  and  $r$  that satisfy this constraint along with the model outcomes generated by these values. In particular, the optimal combination that minimizes total victimization is the combination that allocates 50% of the budget to prevention services and 50% of the budget to long-term aid services. The resulting model output (Figure 2D) shows this optimal combination reduces the total number of victims (first time and repeat) more than prevention alone (Figure 2B) or long-term aid alone (Figure 2C).

**Table 2:** Total costs and total victims over a 10-year period for different combinations of prevention ( $p$ ) and long-term aid ( $r$ ). Combinations were chosen such that total cost does not exceed 3 million dollars.

$r$	$p$	Cost of $r$	Cost of $p$	Total Victims	1st Time	Repeat
0	0.2	\$ -	\$1,500,000	278	223	55
0.05	0.2	\$ 250,000	\$1,500,000	275	221	54
0.1	0.2	\$ 500,000	\$1,500,000	272	220	52
0.15	0.2	\$ 750,000	\$1,500,000	269	218	51
0.2	0.2	\$1,000,000	\$1,500,000	266	217	49
0.25	0.2	\$1,200,000	\$1,500,000	263	215	48
0.3	0.2	\$1,500,000	\$1,500,000	261	215	46
0.35	0.18	\$1,800,000	\$1,200,000	275	227	48
0.4	0.15	\$2,100,000	\$ 900,000	300	250	50
0.45	0.1	\$2,600,000	\$ 400,000	347	291	56

## Conclusions

We created a mathematical model describing human trafficking of underage females. Terms were included in the differential equations to represent aid services for victims and prevention of trafficking. Flexibility is a key aspect of our model. It can be parameterized to represent any population of interest (e.g., urban or rural) over any length of time. Based on the parameters chosen, we found that the use of long-term aid alone or prevention alone was not as effective in reducing victimization over a 10-year period as a multi-faceted approach. The optimal balance of prevention and aid was one that allocated 50% of funding to prevention and 50% of funding to long-term aid services. Organizations like the Human Trafficking Coalition of Pittsburgh can use our model to inform decisions regarding the best use of grant funding and the potential success of prevention programs and long-term aid facilities.

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