Modeling the Effects of Fentanyl and Narcan on the Opioid Epidemic in Allegheny County Using Mathematics

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1DEPARTMENT OF CHEMISTRY AND BIOCHEMISTRY
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Abstract
In collaboration with the Allegheny County Department of Human Services (DHS), we developed a comprehensive mathematical model to describe the opioid epidemic in Allegheny County. The model is a system of differential equations describing how the size of each population class—Susceptible, Prescribed, Addicted, and Recovered—is changing over time. Variables describing the presence of fentanyl (a synthetic opioid) and the use of Narcan (medication used to block the effects of opioids) were included in the model. Model parameters were estimated using data provided by the DHS and reflect the opioid addiction and overdose rates in Allegheny County. Model simulations highlight the impact of fentanyl and Narcan on the annual overdose death rate. Additional results show the extent to which an increase in the availability of Narcan can decrease opioid-related fatalities over time.

Introduction

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Opioid Epidemic in Allegheny County

Figure 3: Mathematical model describing the opioid epidemic in Allegheny County.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values of parameters used in model simulations.</th>
<th>Orange lines describe the observed data and blue lines represent model predictions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
<td>Description</td>
<td>Notes</td>
</tr>
<tr>
<td>𝑆</td>
<td>Proportion of the population that is not using opioids</td>
<td>vary</td>
</tr>
<tr>
<td>𝐴</td>
<td>Proportion of the population that is using opioids as prescribed</td>
<td>vary</td>
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<tr>
<td>𝑅</td>
<td>Proportion of the population that is in treatment for opioid addiction</td>
<td>vary</td>
</tr>
<tr>
<td>𝐴1</td>
<td>Proportion of the population that is using opioids as prescribed</td>
<td>vary</td>
</tr>
<tr>
<td>𝐴2</td>
<td>Proportion of the population that is in treatment for opioid addiction</td>
<td>vary</td>
</tr>
<tr>
<td>𝑁</td>
<td>The population size</td>
<td>used in the simulation</td>
</tr>
<tr>
<td>𝛼</td>
<td>Natural death rate</td>
<td>same for all populations</td>
</tr>
<tr>
<td>𝛽</td>
<td>Natural relapse rate of Non-addict</td>
<td>same for all populations</td>
</tr>
<tr>
<td>𝜀</td>
<td>Induced addiction rate</td>
<td>same for all populations</td>
</tr>
<tr>
<td>𝜉</td>
<td>Prescription rate</td>
<td>same for all populations</td>
</tr>
<tr>
<td>𝜑</td>
<td>Illicit addiction rate based on prescribed opioids</td>
<td>same for all populations</td>
</tr>
<tr>
<td>𝜇</td>
<td>Overdose death rate of addicts involving opioids without Narcan</td>
<td>same for all populations</td>
</tr>
<tr>
<td>𝜈</td>
<td>Overdose death rate of addicts involving opioids with Narcan</td>
<td>same for all populations</td>
</tr>
<tr>
<td>𝜎</td>
<td>Relapse rate of addicts</td>
<td>same for all populations</td>
</tr>
<tr>
<td>𝜒</td>
<td>Availability and distribution of Narcan within the community</td>
<td>same for all populations</td>
</tr>
</tbody>
</table>

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Mathematical Model

Equations (1) - (4) were solved numerically and results were graphed using the software R. The package deSolve was used to solve the equations with the following initial conditions:

Equations and Results

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S(0) = 0.990504, P(0) = 0.0008, A(0) = 0.001136, R(0) = 0.000136

Simulations and Results

Equations (1) - (4) were solved numerically and results were graphed using the software R. The package deSolve was used to solve the equations with the following initial conditions:

S(0) = 0.990504, P(0) = 0.0008, A(0) = 0.001136, R(0) = 0.000136

Conclusions

• Results from model simulations indicate that increasing the availability of Narcan in the community will result in a meaningful reduction in the cumulative number of overdose deaths over a 10-year period.
• However, an increased presence of fentanyl will render Narcan less effective in reducing overdose deaths.
• State and local policymakers, including Governor Wolf, and organizations, such as the Allegheny DHS, can use our model to inform their decisions about the most beneficial way to allocate future funding to combat the opioid epidemic.

Resources


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Figure 6: Graphs display the cumulative number of opioid overdose deaths for three different values of F (percentage of drugs containing fentanyl). In each plot, we assume n = 0.57 (black), 0.62 (green), 0.67 (blue), and 0.72 (red).

Figure 5: Bar plots show model predictions of the annual number of opioid overdose deaths when (A) the effects of fentanyl and Narcan are ignored (F = 0, n = 0); (B) percent of drugs involving fentanyl is increased to 47% in 2015 (F = 0.47) and the impact of Narcan is ignored (n = 0); (C) percent of drugs involving fentanyl is increased to 47% in 2015 (F = 0.47) and the impact of Narcan is increased from n = 0 to n = 0.57 in 2017.